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# DNS of structural vacillation in the transition to geostrophic turbulence

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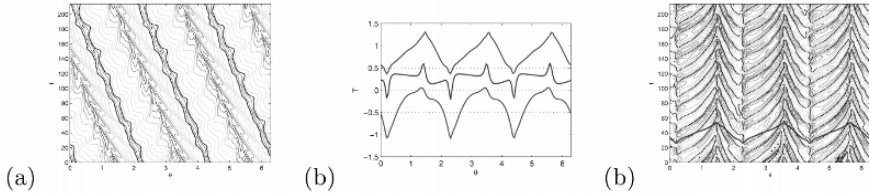
**Summary.** The onset of small-scale fluctuations around a steady convection pattern in a rotating baroclinic annulus filled with air is investigated using Direct Numerical Simulations (DNS). In previous laboratory experiments of baroclinic waves, such fluctuations have been associated with *Structural Vacillation* which is regarded as the first step in the transition to fully-developed geostrophic turbulence. Here we present an analysis which focusses on the small-scale features.

## 1 Introduction

The differentially-heated, rotating cylindrical annulus has proved a fruitful means of studying fully-developed, nonlinear baroclinic instability in the laboratory, e.g. [1]. Transitions within the regular wave regime follow canonical bifurcations to low-dimensional chaos, but disordered flow appears to emerge via a different mechanism involving small-scale secondary instabilities. Not only is the transition to geostrophic turbulence less well understood than those within regular waves, but also the classification and terminology for the weakly turbulent flows is rather vague. Various terms applied include Structural Vacillation or Tilted-Trough Vacillation which both refer to fluctuations by which the wave pattern appears to change its orientation or structure in a roughly periodic fashion. These fluctuations have been explained by the growth of higher order radial mode baroclinic waves, by barotropic instabilities, or by small-scale secondary instabilities or eddies, which lead to erratic modulations of the large-scale pattern. Further within this ‘transition zone’, the gradual and progressive breakdown of the wave pattern leads ultimately to a form of stably-stratified ‘geostrophic turbulence’ [4].

## 2 The numerical model and the case discussed

The model [3] is that of air between two vertical coaxial cylinders of inner radius  $a = 34.8\text{mm}$  and outer radius  $b = 60.2\text{mm}$ , held at constant temper-



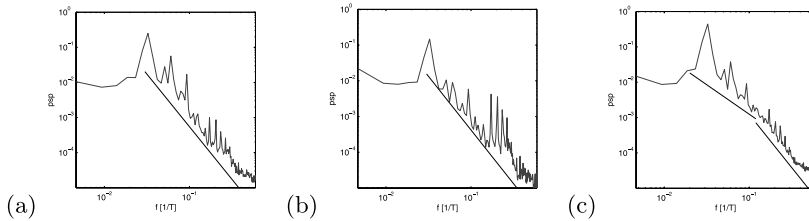
**Fig. 1.** (a) Space-time plot along an azimuthal ring near the warm outer wall. Contour lines show the temperature with a contour interval of 0.1. (b) Time-averaged temperature profiles at the three radial positions from the temperature profiles shifted to a co-rotating frame. (c) Temperature residuals near the outer wall in a space-time plot with temperature contours 0.025. The time-averaged profile is superimposed.

atures, here  $T_b = 308\text{K}$  and  $T_a = 278\text{K}$ , and two horizontal insulating rigid lids separated by a distance  $d$ . The cavity rotates around the central axis, here  $\Omega = 52\text{rad/s}$ . The model equations are solved using a pseudo-spectral collocation-Chebyshev in the radial and vertical, and Fourier method in the azimuthal direction with varying model resolutions, here  $108 \times 108 \times 128$ . The time integration, based on a combination of Adams-Bashforth and Backward Differentiation Formula schemes is semi-implicit and second order accurate.

### 3 Results

This flow, and the transition to it, is presented elsewhere [5]. This paper focusses on the small-scale structures by an alternative method to filter out the large-scale flow, applied to azimuthal temperature profiles at mid-height and three radial positions, at mid-radius and around 15% from each wall. As the solution was characterised by an almost steady wave drifting along the channel on which small-scale fluctuations were evident (Fig. 1a for the profiles near the outer wall), the results could be transformed to a frame moving with the wave structure. From this, the time-averaged spatial structure could be obtained (Fig. 1b for all radial positions), which could then be subtracted from the standing wave to obtain time series of the residual temperature profiles (Fig. 1c). The space-time plots of the residuals highlight that small perturbations are emitted fairly regularly (but not periodically) near the warm outer wall from the cold jet which brings cold air towards that wall, and that these perturbations travel, initially fairly rapidly, in the azimuthal direction but then slow down as they approach the hot jet, which originates near this wall and takes fluid towards the inner wall. The case is similar near the cold inner wall with perturbations originating from the incoming hot jet. In the centre of the gap, however, the fluctuations appear to be localised to the jets.

Spatially averaged power spectra from the temperature residuals at the three profiles are shown in Figure 2. The main frequency ( $\sim 0.03$ ) is that of



**Fig. 2.** Power spectra from the time series of the temperature spectra, spatially averaged over each profile: (a) near the inner wall, (b) at mid-radius, and (c) near the outer wall. The frequency is in units of the inverse of the reference time scale, i.e.  $2\Omega$ . All three show a line  $\sim f^{-3}$ ; (c) also shows a line  $\sim f^{-5/3}$  at lower frequencies.

the emission of the perturbations. The spectrum falls off at all radial positions, with some distinctive peaks over the general decay. While the decay at higher frequencies is largely consistent with a  $f^{-3}$  law at all positions, a slight flattening in the frequency range between that of the main perturbation and about 0.1 can be observed near the outer wall. The behaviour of the spectrum in that range appears to be closer to a  $f^{-5/3}$  law.

The spectral evidence suggests that the flow investigated here consists of a fairly steady large-scale convection pattern in the form of three pairs of hot and cold radial jets. From those jets, smaller perturbations are emitted at relatively regular intervals where a jet approaches a wall. The overall flow responds in a cascade of faster (and smaller) fluctuations which appear consistent with two-dimensional, quasi-geostrophic turbulence over a wide range of frequencies similar to results of DNS of geostrophic turbulence. Recently, similar energy spectra were observed for geostrophic turbulence in a square box [2, 6].

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